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DEFORMATION OF A VISCOPLASTIC MEDIUM UP TO FRACTURE

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A general mathematical model is formulated for the unsteady deformation of an isotropic viscoplastic medium, together with the necessary and sufficient conditions for loss of continuity. Using the tension of a rectilinear strip and of a round bar up to fracture as examples, a matrix is determined for identifying metals and their alloys with respect to operational reliability.

Mathematical model

Consider, in a system of three-dimensional coordinates x and time t, a region $\Omega(x,t)$ with its own changing boundary $\Gamma(t)$. We postulate that in the region $\Omega(x,t)$ an unsteady, irrotational deformation of a continuous isotropic medium takes place. The stress tensor $G = G(x,t)$, the velocity vector $v = v(x,t)$ and the density of the medium $\rho = \rho(x,t)$ in $\Omega(x,t)$ then satisfy the classical system of equations [1]:

$$\rho \cdot (\partial v/\partial t + v \cdot \nabla v) = \operatorname{div} G \qquad (1)$$

$$\partial \rho/\partial t + \operatorname{div} (\rho v) = 0 \quad (2)$$

$$Q(x, t) = Q(\rho) \quad (3)$$

Here (1) is the equation of unsteady deformation of a continuous medium outside a field of external body forces; (2) is the condition of continuity (indivisibility) of the material under consideration; (3) is the equation of state of the medium, given as a barotropic function $Q(\rho)$. The linear operators ∇ and div are, respectively, the gradient and the divergence of a tensor (vector) quantity.

Let $F(x, t) = 0$ denote the equation of the boundary $\Gamma(t)$ of a particular volume of the medium $\Omega(x, t)$. On that boundary the kinematic and dynamic conditions must hold, respectively:

$$\partial F/\partial t + v \cdot \nabla F = 0 \quad (4)$$

$$\sigma \cdot n = P \quad (5)$$

where n is the normal vector to the boundary $F(x, t)$; $P(x, t)$ is the value of the prescribed pressure at the boundary.

The kinematic condition (4) at the boundary expresses the fact that this surface consists of the same particles of the medium. In other words, the normal component of the velocity of the medium at the boundary coincides with the velocity of displacement of the surface of the medium in the direction of the normal. The dynamic condition (5) is the Cauchy relation (continuity) for the stresses at the boundary of the deforming region $\Omega(x,t)$.

As a rule, when applied problems in metal forming are considered, or when components of power plants are studied in service, three types of boundary occur: a fixed solid boundary F_1, a moving solid boundary F_2, and a free boundary F_3. Equation (5) holds for all of these boundaries. In the first case F_1 is a known function, $\partial F_1/\partial t = 0$ and $v = 0$ at the boundary (the no-slip condition). In the second case F_2 is known and the condition of impermeability of the solid boundary is satisfied. In the third case F_3 is known only at the initial instant $t = 0$; for $t > 0$ the boundary changes and is determined from the solution of the whole system of relations (1)–(5).

Initial conditions (at $t = 0$) must be added to the system of relations (1)–(5):

$$v_0 = v(x,0);\ \rho_0 = \rho(x,0);\ \Omega_0 = \Omega(x,0);\ F_0 = F(x,0) \quad (6)$$

Equations (1)–(6) define a closed mathematical model describing the irrotational unsteady deformation of a continuous medium in a region whose boundary is prescribed at the initial instant. There is, however, no specification of the medium here. Since this article is devoted to the deformation of metals under all existing loads, including dynamic ones, we adopt the model of a viscoplastic medium. A more detailed argument for this approach is given in [2].

In the work of A. A. Ilyushin [3], two fundamental hypotheses of a viscoplastic medium are formulated:

— the directions of the maximum slip rates coincide with the directions of the maximum stresses at every point of the continuous medium;

— the maximum shear stress during deformation of the medium is always greater than some constant and is a function of the maximum slip rate.

Let the functions $\Pi(x,t)$ and $H(x,t)$ denote, respectively, the invariants of the intensity of the shear stresses and of the shear strain rates. Then, according to the hypotheses, for a viscoplastic material we have the linear relation

$$\Pi(x,t) = \sigma_{02} + \mu\, H(x,t) \quad (7)$$

where σ_{02} is the plastic constant characterising the strength property of the medium. In what follows it is taken equal to the yield strength of the material in the calculations, with the dimension [MPa]; μ is the dynamic viscosity coefficient of the medium [Pa·s], which characterises internal friction and the loading history in time.

Note that in the case ρ = const, $\mu = 0$, $\sigma_{02} > 0$ the system of relations (1)–(7) describes the classical model of deformation of a material in the ideal-plasticity scheme [4]. For ρ = const, $\mu > 0$, $\sigma_{02} = 0$ it is the model of a viscous fluid [5]. When ρ = const, μ =

0, $\sigma_{02} = 0$ it is the model of an ideal fluid [6].

Clearly, the medium under consideration, possessing strength properties, cannot deform indefinitely without a violation of continuity. Under certain conditions local or systemic fracture of the material occurs under the applied loads. The concept of fracture — loss of continuity of the medium — is a complex problem lying at the junction of solid-state physics, continuum mechanics and materials science. Fracture mechanics has now become a comprehensive science of the solid body, in which a large number of general and particular results have been obtained in various model formulations (see, for example, [7–9]). For the mathematical model under consideration we adopt the sufficiently well known and practically confirmed energy criterion of fracture due to A. G. Ivanov [10, 11]. Hence we have the necessary condition of fracture in the form of the integral equation

$$\int q(t,V)\mathrm{d}V = \int A_* \mathrm{d}S \quad (8)$$

Here $V(x,t)$ is the volume of the part of the medium from which the elastic energy necessary for fracture has been removed, $q(t,V)$ is the density of elastic energy released on unloading by the elastic wave, and A_* is the specific work expended on the formation of the fracture surface S. The parameter A_* corresponds to the effective energy of dynamic fracture and is a characteristic of the material. According to [12], the density of elastic energy $q(t,V)$ is estimated by the formula

$$q(t,V) = \Pi^2(x,t) / 2E \qquad (9)$$

where E is Young's modulus.

Clearly, if there is a necessary condition of fracture then, by common sense, there must also be a sufficient one. We supplement the model under consideration with the following statement: when one of the components of the stress tensor $G(x,t)$ in the medium reaches the value of the ultimate tensile strength, loss of continuity occurs in the material:

$$|G(x,t)| \geq \sigma_b \quad (10)$$

where σ_b is the ultimate tensile strength, a characteristic of the material.

The internal structure of continuously deforming real bodies is usually not in stable thermodynamic equilibrium. In fact, the further from such an equilibrium state, the more intense the deformation [13]. Constructing a mathematical model to describe a particular physical process is, as a rule, possible in different ways. The main criteria here are that the laws of thermodynamics be satisfied, from which it follows that the rate of dissipation of mechanical energy per unit volume must always be positive [4]. Let $W(x,t)$ denote the dissipative function. Then

$$W(x,t) = \sigma_{ij} \cdot \varepsilon_{ij}, \quad W > 0 \ (11)$$

where σ_{ij} and ε_{ij} are, respectively, the components of the stress tensor and of the strain-rate tensor.

Thus the system of relations (1)–(11) is a closed mathematical model, with boundary and initial conditions, describing the unsteady deformation of an isotropic viscoplastic medium up to fracture without allowance for a field of external body forces, where the free boundary of the deforming region is known only at the initial instant.

For all the fundamental nature of the formulation of the problem, it is possible, under certain assumptions, to determine a class of exact analytical solutions of the model (1)–(11) for the simplest regions: a rectilinear strip [14], a round bar [15, 16], a ring [17], a cylinder [18] and a sphere [19, 20].

The process of unsteady deformation of a bounded volume of a viscoplastic medium with prescribed loads at the boundary proceeds up to a certain time $t_* > 0$ without loss of continuity. During this time a microcrack is nucleated at the microlevel and grows to a critical size, reaching the boundary. Then, for $t > t_*$, the crack or system of cracks propagates intensively through the material, up to the separation of the deforming region into individual parts. In essence, during the time $t \in (0, t_*)$ the fracture process passes from the microlevel to the macrolevel (for $t > t_*$). Under dynamic loads, compression and tension waves are formed in the deformation region. It is known that wave processes in metals — for example under explosive loading — are substantially shorter in time than deformation processes.

The following hypotheses were therefore adopted:

— elastic unloading waves from the crack surface propagate in both directions at the velocity of longitudinal sound waves [21];

— the process of unsteady deformation of the viscoplastic medium in the region under consideration proceeds without allowance for the interaction of shock waves in the material ($t > t_*$).

The solutions obtained for the strip, bar, ring, cylinder and sphere revealed a regularity that is important in practice: in every case a viscoplastic material exhibits a ductility peak as a function of the strain rate, extending as far as superplasticity [22]. This fact is confirmed by experimental data (see, for example, [23]).

Matrix for the identification of metals

To date, the viscoplastic-medium model has found no wide practical application in fracture mechanics — mainly through ignorance of the specific values of the parameters μ and A_* when design and technological problems are solved, for example in metal forming. Determining them is a topical scientific subject.

To this end, consider the method long known in materials science for determining the mechanical properties of metals: the ultimate tensile strength σ_b, the yield strength σ_{02} and the elongation δ_5. These values are determined by uniaxial tension of flat or round specimens to fracture, at a constant strain rate ($\dot{e}$ = const) and a fixed test temperature T and constant density (ρ = const). The process is strictly regulated, in accordance with the standards GOST 1497-84 and GOST 10006-80.

Hence, taking into account the simplifications permitted by those standards when the mathematical model is solved for the strip [14] and the bar [16] — ρ = const, $\dot{e}$ = const under uniaxial tension of the specimens — we obtain for the components of the stress tensor, in the case of the strip and of the bar respectively,

$$\sigma_{xx} = \sigma_{02} + 2\mu\dot{e}, \quad \sigma_z = \sigma_{02} + 3\mu\dot{e} \quad (12)$$

for a linear velocity field, where $H = 2\dot{e}$ for the strip and $H = 3\dot{e}$ for the bar.

Formulas (7)–(10) and (12), together with the hypothesis of unloading by an elastic wave from the crack surface at the velocity of sound $c = (E/\rho)^{1/2}$, then make it possible to obtain the following relations:

for tension of a flat specimen

$$\mu = (\sigma_b - \sigma_{02}) / 2\dot{e}, \quad A_* = (c\delta_5\sigma_{02}{}^2) / (2E\dot{e}) \cdot (1 + 2\mu\dot{e}/\sigma_{02})^2 \ (13)$$

for tension of a round specimen

$$\mu = (\sigma_b - \sigma_{02}) / 3\dot{e}, \quad A_* = (c\delta_5\sigma_{02}{}^2) / (2E\dot{e}) \cdot (1 + 3\mu\dot{e}/\sigma_{02})^2 \ (14)$$

By the definition of the parameters μ and A_*, the higher their values for the material under consideration, the higher its operational reliability. What is meant here is the concept as a whole: strength, crack resistance and endurance limit. We thus arrive at the definition of the matrix for identifying metals and their alloys in the form

$$(\sigma_b, \sigma_{02}, \delta_5, \mu, A_*, \dot{e}, T) \quad (15)$$

The values σ_b, σ_{02} and δ_5 are determined experimentally in accordance with GOST 1497-84 and GOST 10006-80; the parameters μ and A_* are found by calculation from formulas (13)–(14) at a fixed strain rate $\dot{e}$ and test temperature T.

Note that the normative and technical documentation for rolled metal products currently uses the matrix

$$(\sigma_b, \sigma_{02}, \delta_5, A_k, K_{1c}, T) \quad (16)$$

where A_k is the impact toughness, determined in accordance with GOST 9454-78, and K_{1c} is the crack-resistance coefficient, GOST 25.506-85. Estimating the parameters A_k and K_{1c} by a calculation-experimental method has a number of fundamental limitations (see, for example, [24]). The main difference between matrices (15) and (16) is the concept of determining the parameters A_k and K_{1c} at small deformations of the material, close to the model of an elastoplastic medium. It is known that real metals and their alloys have considerable ductility, δ_5 = (20–70)%, particularly under dynamic loads. For example, an aluminium ring (AD1) of diameter 52 mm and wall thickness 9 mm, under the initiation of a cylindrical explosive charge (TG 50/50) weighing 8 g, expands up to fracture with a limiting deformation $\delta_5 \approx 43\%$. In standard tension of a flat specimen of alloy AD1 in accordance with GOST 1497-84 we have $\delta_5 \approx 28\%$. Figure 1 shows a radiograph of the expansion of an aluminium ring (52 × 9 × 10 mm) with grooves at the instants 0, 25, 32 and 40 µs. The photographs show substantial deformation of the ring material ($\delta_5 \approx 45\%$) before the ring fractured along the inner groove [25].

The performance of matrix (15) is considered using the example of an assessment of the operational reliability of steel capillary tubes. Because of the small geometric dimensions of the specimens, the standards for impact toughness (GOST 9454-78) and crack resistance (GOST 25.506-85) do not work.

Table 1 gives the mechanical properties of a 3.0 × 0.5 mm capillary tube of corrosion-resistant steel 12Kh18N10T (GOST 14162-79) in the initial state, after heat treatment, and the size 3.0 × 0.6 mm obtained after drawing a 5.0 × 0.5 mm tube without heat treatment. The tensile test specimens were tubes 3.0 × 0.5 × 200 mm. The parameters μ and A_* were calculated from formulas (14) at E = 210 GPa, ρ = 7.92 g/cm^3, c = 5156 m/s. In accordance with GOST 1497-84 the tensile strain rate of the specimens is regulated and fixed at $\dot{e} = 0.25 \cdot 10^{-2}$ 1/s. The calculated data for μ and A_* are given in Table 1.

Table 2 summarises the mechanical properties of capillary tubes of heat-resistant steel EP648-VI at +20 °C, together with the values of μ and A_* calculated from formulas (14). The parameters E, c, ρ and $\dot{e}$ were taken as in the first example. From Tables 1 and 2 it follows that capillary tubes in the cold-worked condition (without heat treatment) have a low level of operational reliability. At the same time these tubes possess the highest strength. This tendency is not valid in every case. For example, in assessing the endurance limit of titanium alloys [26], or in considering the effect of different types of heat treatment on the strength of steels [27], there are cases in which greater strength of the metal corresponds to high operational reliability.

The calculation-experimental method of identifying metals presented here, in the form of the genome-forming matrix (15), where the dynamic viscosity coefficient μ and the specific energy A_* are determined by simple, physically justified mathematical formulas (13)–(14), has practical promise.

The article is dedicated to the memory of Professor Vladimir Mikhailovich Kuznetsov, Dr. Sc. (Phys.-Math.) (1930–1984), my scientific supervisor for my university diploma (Novosibirsk State University, 1972) and for my candidate's dissertation (1975).

Table 1. Mechanical properties of a 3.0 × 0.5 mm capillary tube of steel 12Kh18N10T at +20 °C.

No.	Drawing route	σb (MPa)	σ02 (MPa)	δ5 (%)	μ (GPa·s)	A* (GJ/m²)
1	3.0 × 0.5 mm (initial, after heat treatment)	760	410	49	46.7	1390
2	5.0 × 0.5 mm → 3.0 × 0.5 mm (67% deformation)	1168	1062	9	14.1	603

Table 2. Mechanical properties of tubes of alloy EP648-VI (KhN50VMTYuB) at +20 °C.

No.	Tube size	σb (MPa)	σ02 (MPa)	δ5 (%)	μ (GPa·s)	A* (GJ/m²)
1	5 × 0.3 mm	880	522	41	47.7	1434
2	2.5 × 0.3 mm	1013	596	37	55.6	1716
3	2.5 × 0.3 mm (without heat treatment)	1506	1278	5	30.4	512

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